CITY UNIVERSITY LONDON

# Microeconomics Pre-sessional September 2016

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# Organisation of the Microeconomics Pre-sessional

| | |
|---|---|
| • Introduction | 10:00-10:30 |
| • Demand and Supply | 10:30-11:10 |
| | *Break* |
| • Consumer Theory | 11:25-13:00 |
| | *Lunch Break* |
| • Problems – Refreshing by Doing | 14:00-14:30 |
| • Theory of the Firm | 14:30 -15:30 |
| | *Break* |
| – Problems – Refreshing by Doing | 15:45 -16:30 |

# Outline

1. The production function
   – Marginal and average product
   – Isoquants
   – Marginal Rate of Technical Substitution
   – Elasticity of substitution
   – Returns to scale
2. Cost and cost minimization
3. Cost functions
   – Long-run vs. short-run

# 1. Production function

- The production function tells us the *maximum* possible output that can be attained by the firm for any given quantity of inputs.

$$Q = f(L,K,\ldots)$$

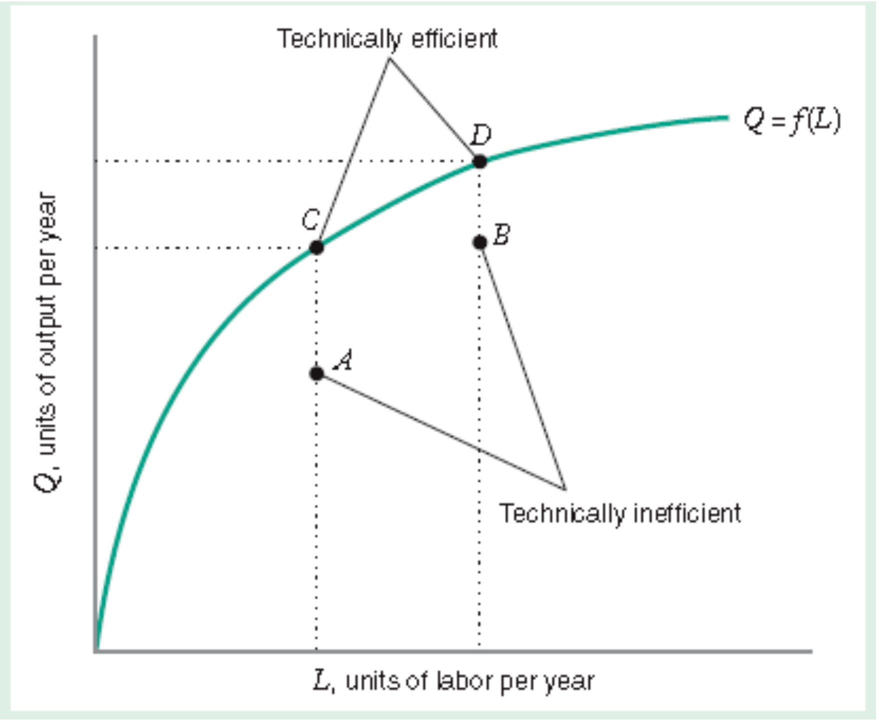
- Definitions:
  - INPUTS (or factors of production): Productive resources, such as **labor** (L) and **capital equipment** (K), that firms use to manufacture goods and services
  - OUTPUT: Amount of goods and services produced by the firm
  - Technology determines the quantity of output that is feasible to attain for a given set of inputs.

# 1. Production function

Production set: set of all technically feasible combinations of inputs and outputs

A technically efficient firm is attaining the maximum possible output from its inputs (using whatever technology is appropriate)

Example: Q = f(L)




f(L) is the total product function

# 1. Production function

**Marginal product** of an input is the change in output that results from a small change in an input *holding the levels of all other inputs constant*.

Example: $Q = f(L,K)$

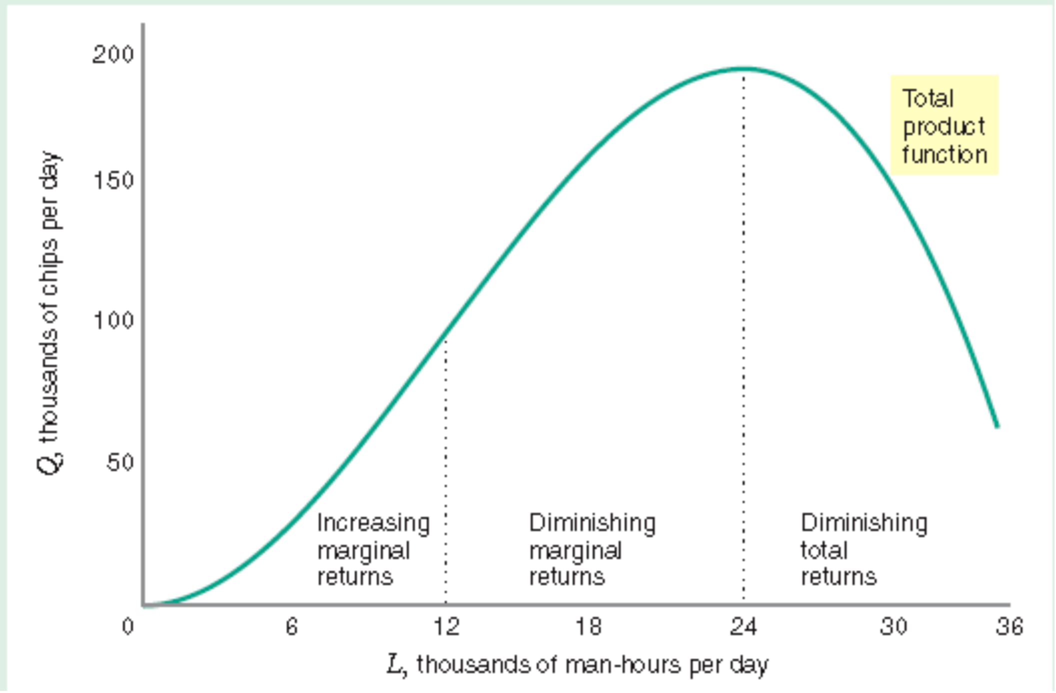
$$MP_L = \frac{\text{change in total product}}{\text{change in quantity of labour}} = \frac{\Delta Q}{\Delta L}$$ (holding *K* constant)

$$MP_K = \frac{\text{change in total product}}{\text{change in quantity of capital}} = \frac{\Delta Q}{\Delta K}$$ (holding *L* constant)

# 1. Production function

The **law of diminishing marginal returns** states that marginal products (eventually) decline as the quantity used of a single input increases.

<u>Example:</u> $Q = f(L)$



# 1. Production function

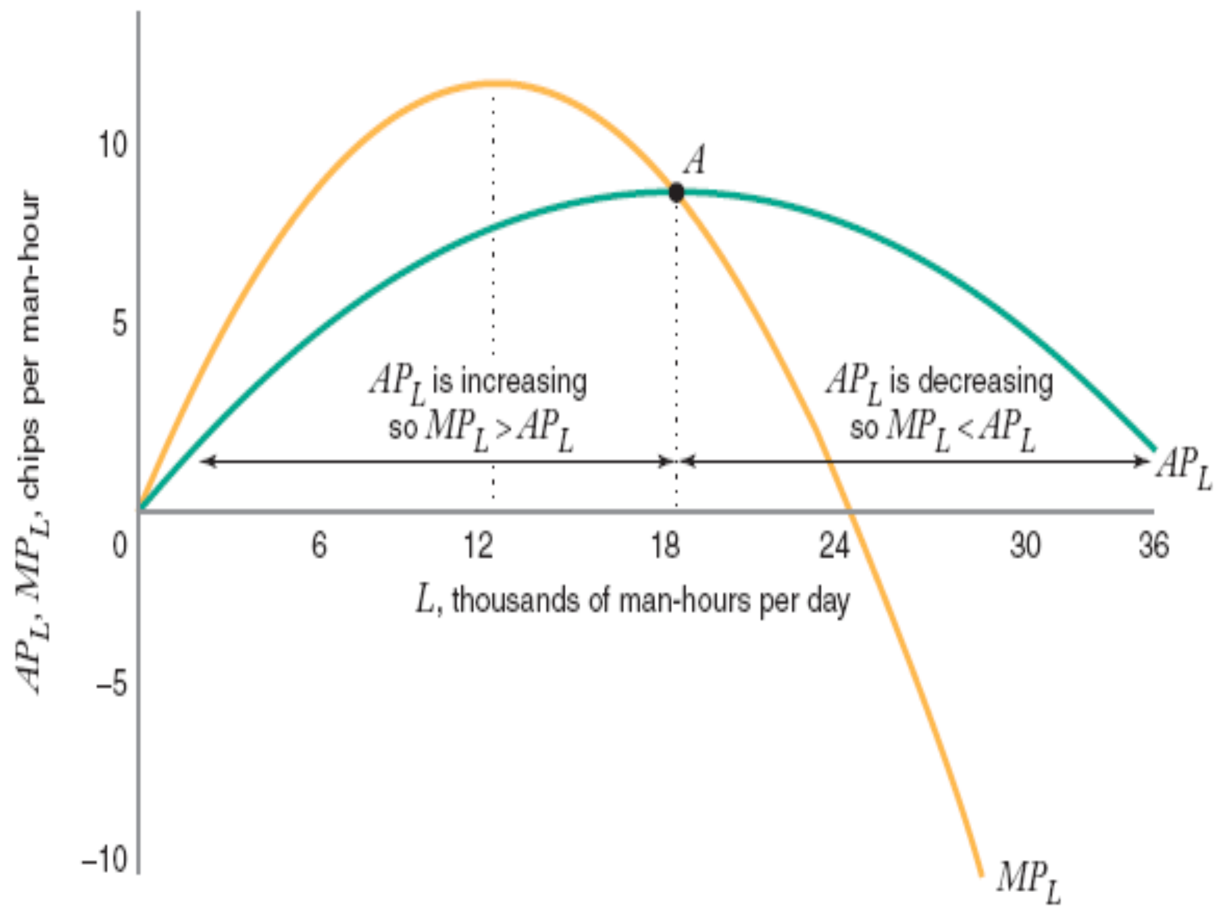
**Average product** of an input is equal to the average amount of oputput per unit of input.

$$Q = f(L,K)$$

$$AP_L = \frac{\text{total product}}{\text{quantity of labour}} = \frac{Q}{L}$$

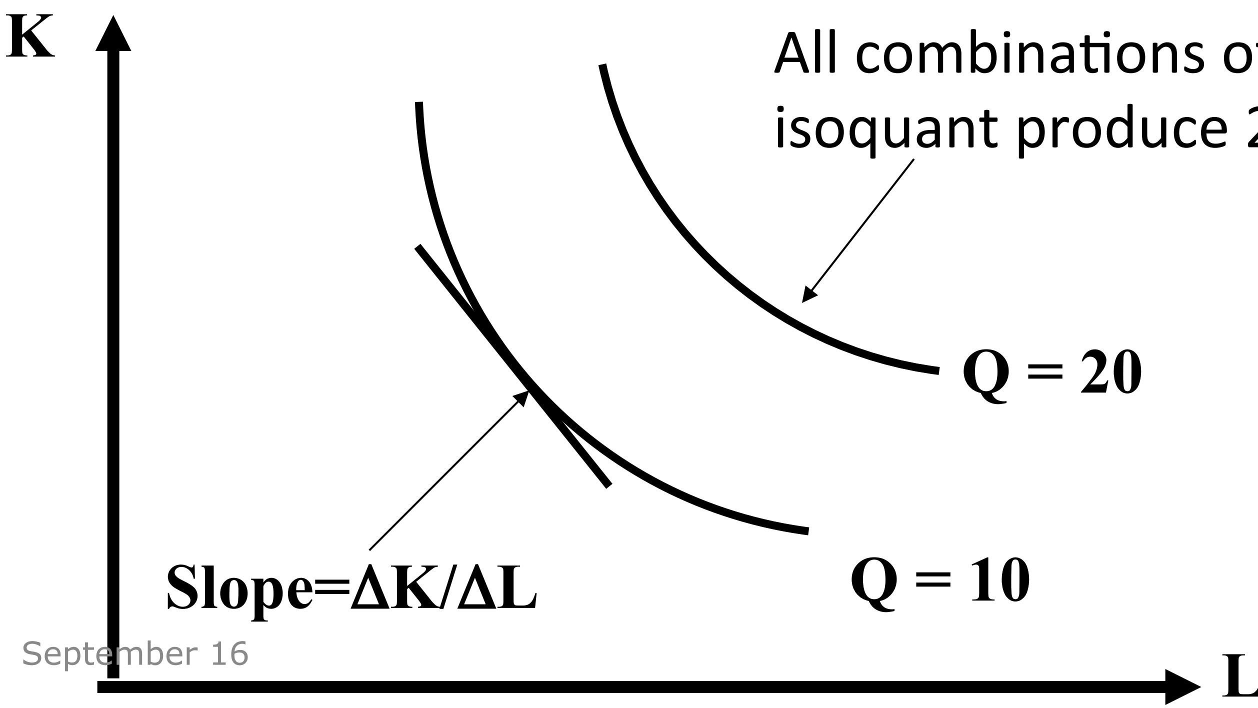
$$AP_K = \frac{\text{total product}}{\text{quantity of capital}} = \frac{Q}{K}$$

# 1. Production function



# Isoquants

- **Isoquants:** combinations of inputs that produce the same level of output
  - Two-input example: Q=q(K,L)
  - Substitution between inputs (i.e. physicians and nurses)
  - Different isoquants represent different output levels



# Isoquants

- Marginal rate of technical substitution measures the rate at which the quantity of an input, K, can be decreased, for every one-unit increase in the quantity of another input, L, holding the quantity of output constant

$$MRTS_{L,K} = -\Delta K/\Delta L$$

$$MRTS_{L,K} = MP_L/MP_K$$

# Isoquants

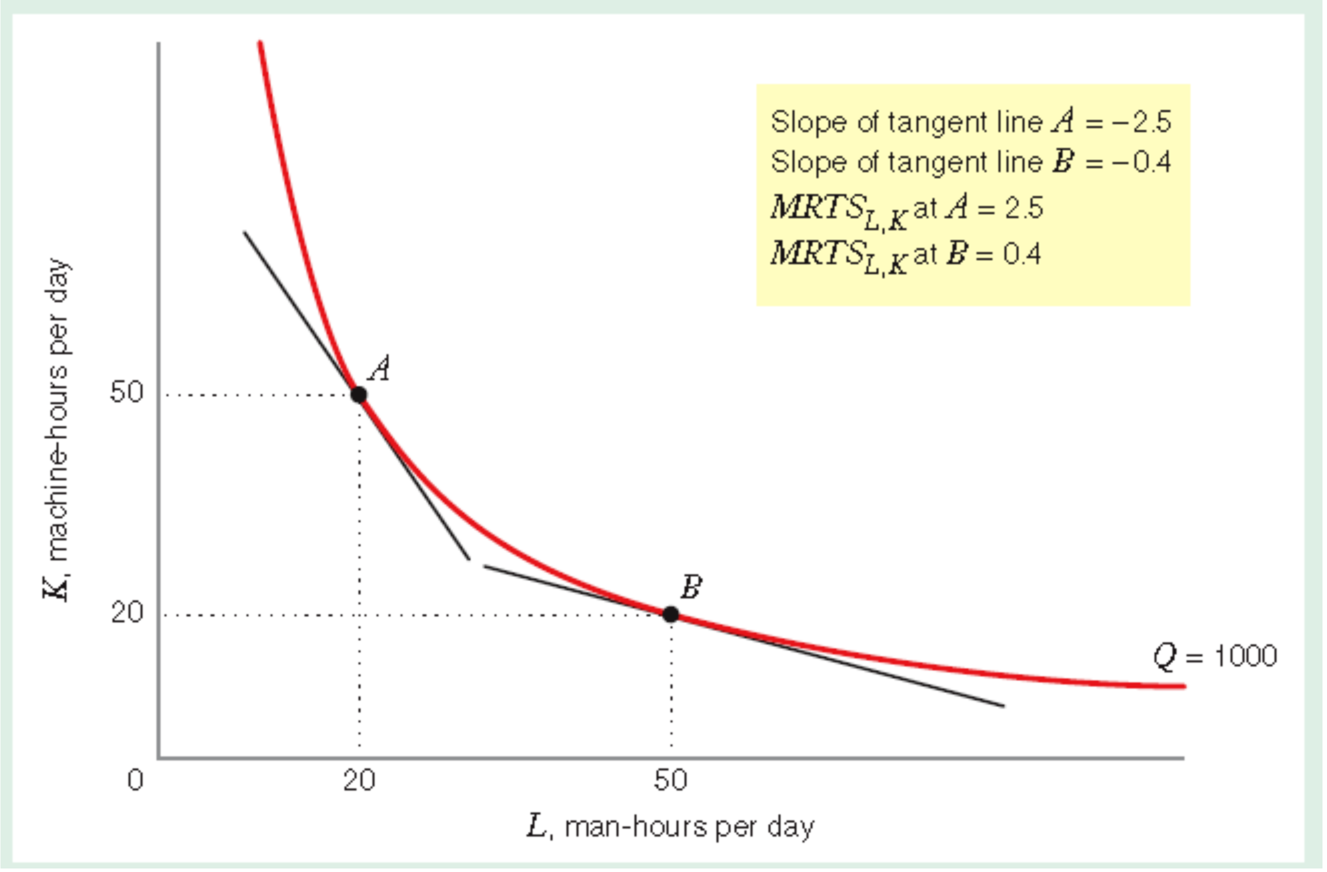
- Marginal rate of technical substitution measures the rate at which the quantity of an input, K, can be increased, for every one-unit decrease in the quantity of another input, L, holding the quantity of output constant

$$MRTS_{L,K} = -\Delta K/\Delta L$$

$$MRTS_{L,K} = MP_L/MP_K$$

- Example:

$$MRTS_{L,K} = -\Delta K/\Delta L \qquad MRTS_{L,K} = MP_L/MP_K$$



# Elasticity of substitution

- The elasticity of substitution measures how easy it is for a firm to substitute one input, L, for other input, K, as we move along an isoquant (holding other inputs and the quantity of output constant)

$$\sigma = \frac{\%\,\text{change in capital-labour ratio}}{\%\,\text{change in } MRTS_{L,K}} = \frac{\%\ \Delta(K/L)}{\%\ \Delta MRTS_{L,K}}$$

# Special production functions

- Cobb-Douglas
- Linear (perfect substitutes)
- Perfect complements

(similar to special utility functions)

# Returns to scale

*How much will output increase when ALL inputs increase by a particular (percentage) amount?*

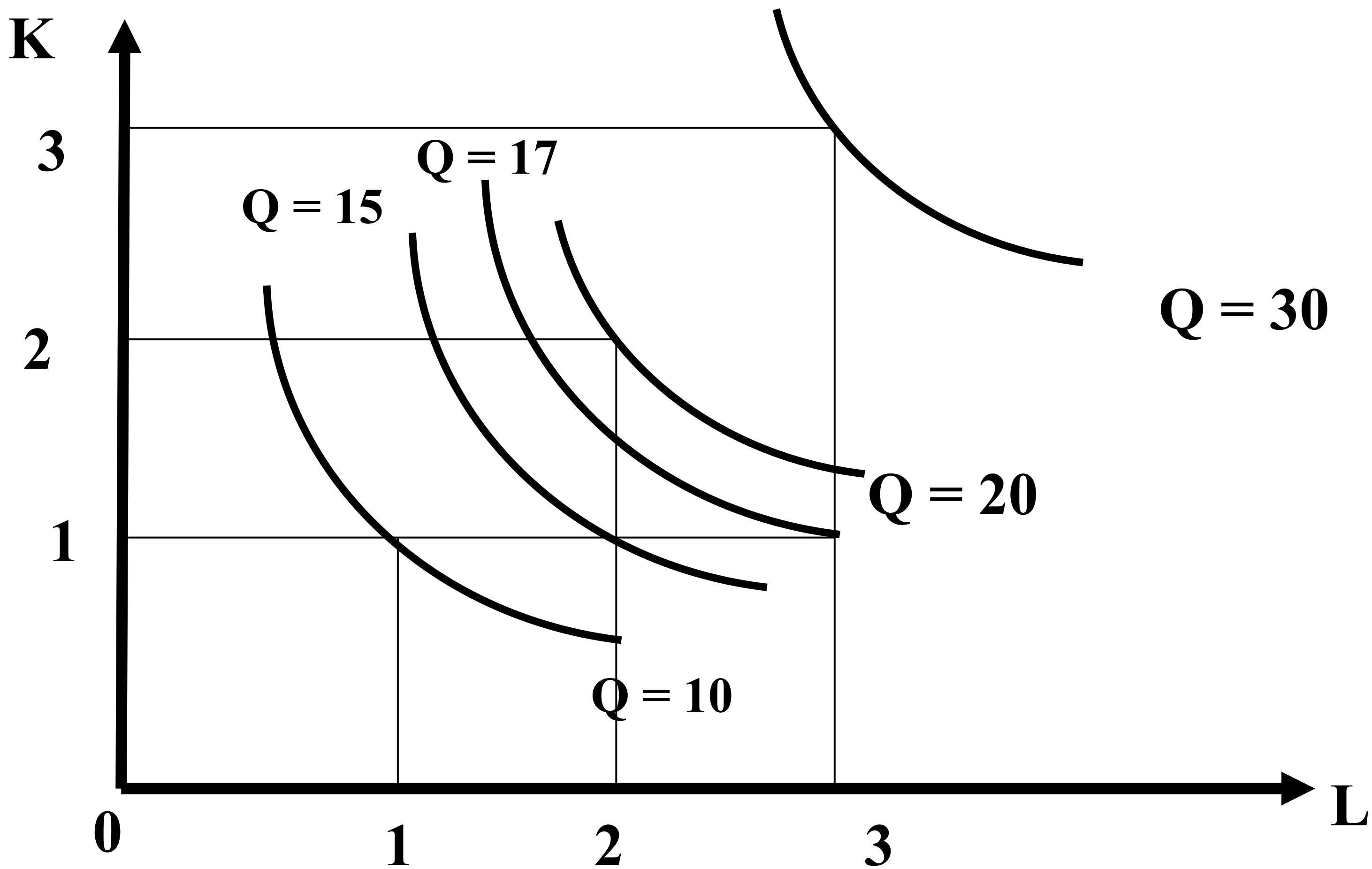
$$RTS = \frac{\%\ \Delta Q}{\%\ \Delta(\text{all inputs})}$$

# Returns to scale

- Increasing returns to scale → 1% increase in all inputs results in a greater than 1% increase in output, then the production function exhibits increasing returns to scale.
- Constant returns to scale → 1% increase in all inputs results in exactly a 1% increase in output, then the production function exhibits.
- Decreasing returns to scale → 1% increase in all inputs results in a less than 1% increase in output, then the production function exhibits.

$$RTS = \frac{\%\ \Delta Q}{\%\ \Delta(\text{all inputs})}$$

# Returns to scale -example



# Differences between production and utility function

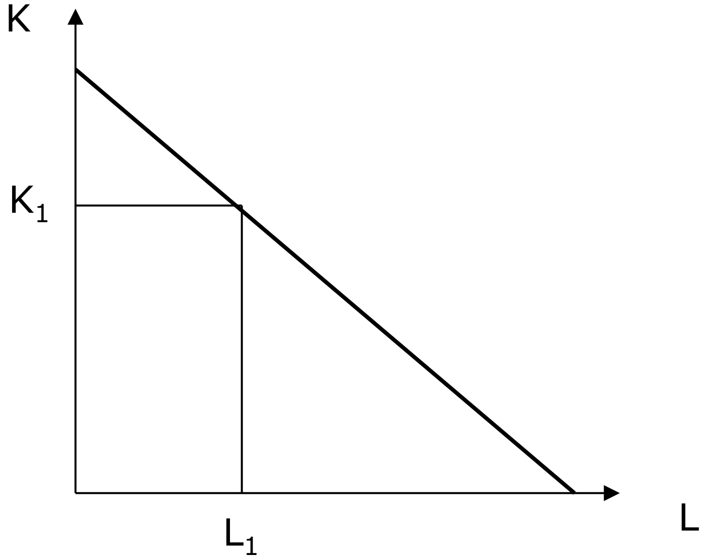
| Production Function | Utility Function |
|---|---|
| Output from input | Utility level from purchases |
| Derived from technologies | Derived from preferences |
| Cardinal (given amount of inputs yields a unique and specific amount of output) | Ordinal |
| Marginal product | Marginal utility |
| Isoquant (all possible combinations of inputs that just suffice to profuce a given amount of output) | Indifference curve |
| Marginal rate of technical substitution | Marginal rate of substitution |

# 2. Costs

- Cost of all inputs used in the production function
  - Input bundle $(x_1, x_2, \ldots, x_n)$ where $x_i$ is the quantity of the input used
  - Input prices $(p_1, p_2, \ldots, p_n)$
  - Total cost of input vector
    - $C = p_1x_1 + p_2x_2 + \ldots + p_n x_n$
  - Two-input example:
    - $C = p_1x_1 + p_2x_2$

# 2. Costs

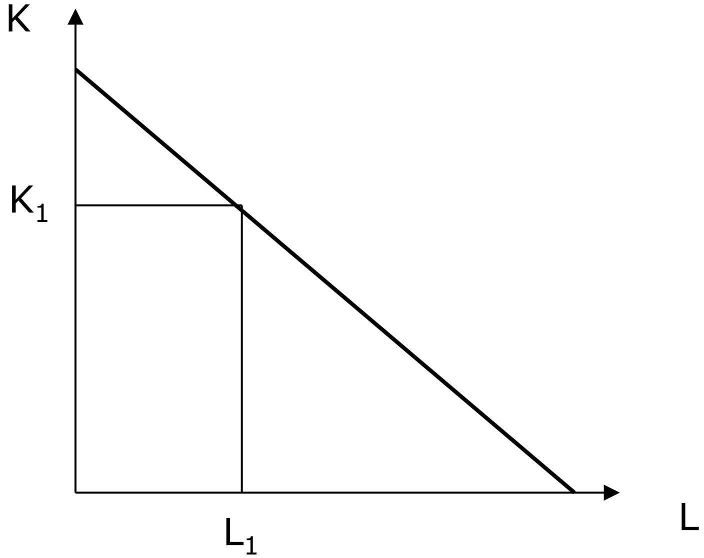
- $C = p_L L + p_K K$ represents an isocost line
- It captures input-cost relationship




- Represents all combinations of inputs that the producer can pay for a given budget (**C**)

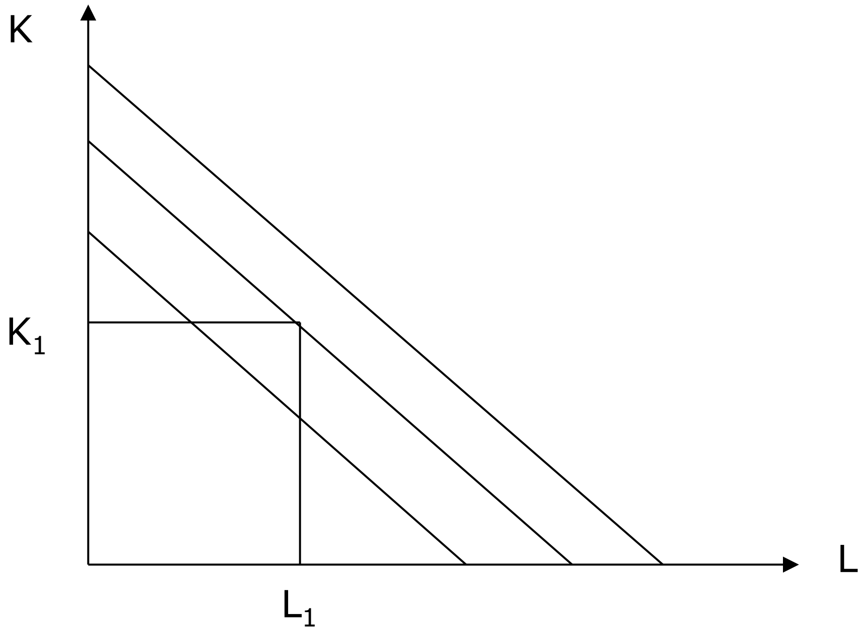
# 2. Costs

- $C = p_L L + p_K K$ represents an isocost line
- Slope of isocost:



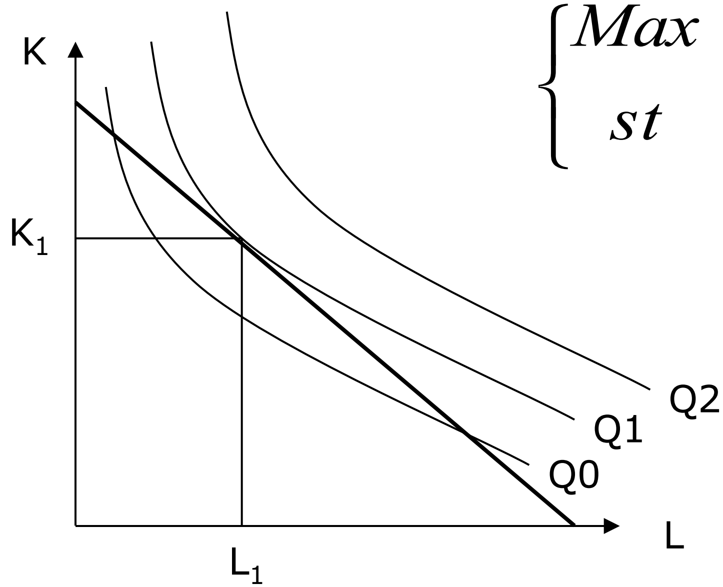
# 2. Costs

- $C = p_L L + p_K K$ represents an isocost line
- Isocost will shift outwards the greater the cost of production inputs (total cost C ), prices constant



# 2. Costs

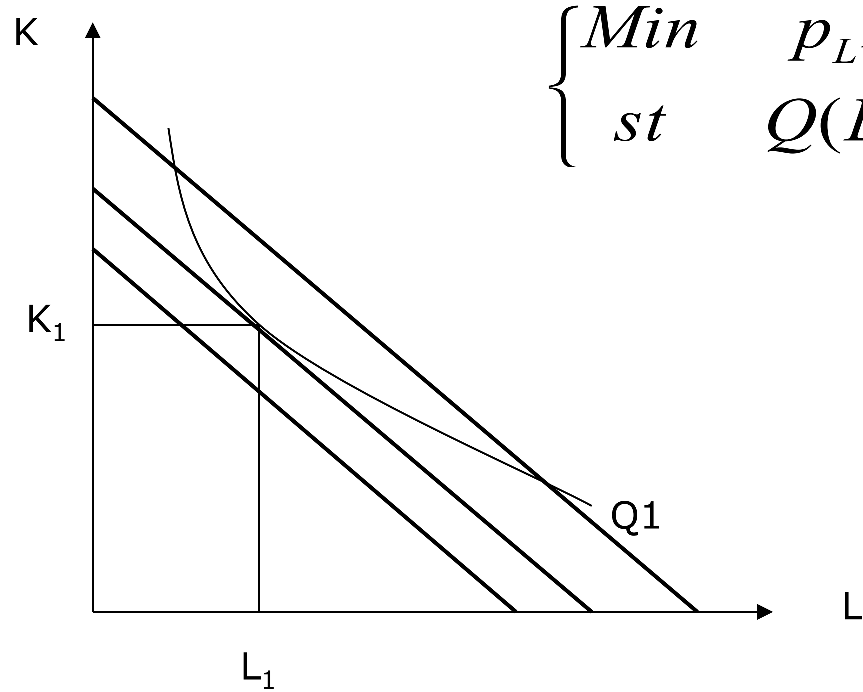
- Output-cost relationship (long-run!)
- Economic efficiency:
  - Max output for specific budget C




$$\begin{Bmatrix} Max & Q(L,K) \\ st & p_L L + p_k K = C \end{Bmatrix}$$

# 2. Costs

- Economic efficiency:
  - Min cost given a level of output




$$\begin{cases} Min & p_L L + p_k K \\ st & Q(L,K) = Q \end{cases}$$

# 2. Costs

- Max/Min problem have same solution

$$\begin{Bmatrix} Max & Q(L,K) \\ st & p_L L + p_k K = C \end{Bmatrix} \qquad \begin{Bmatrix} Min & p_L L + p_k K \\ st & Q(L,K) = Q \end{Bmatrix}$$

Solution :

$$L^*(p_L, p_k, C)$$
$$K^*(p_L, p_k, C)$$

Solution :

$$L^*(p_L, p_k, Q)$$
$$K^*(p_L, p_k, Q)$$

# 2. Costs

(Interior solutions)

- Max/Min problem have same solution

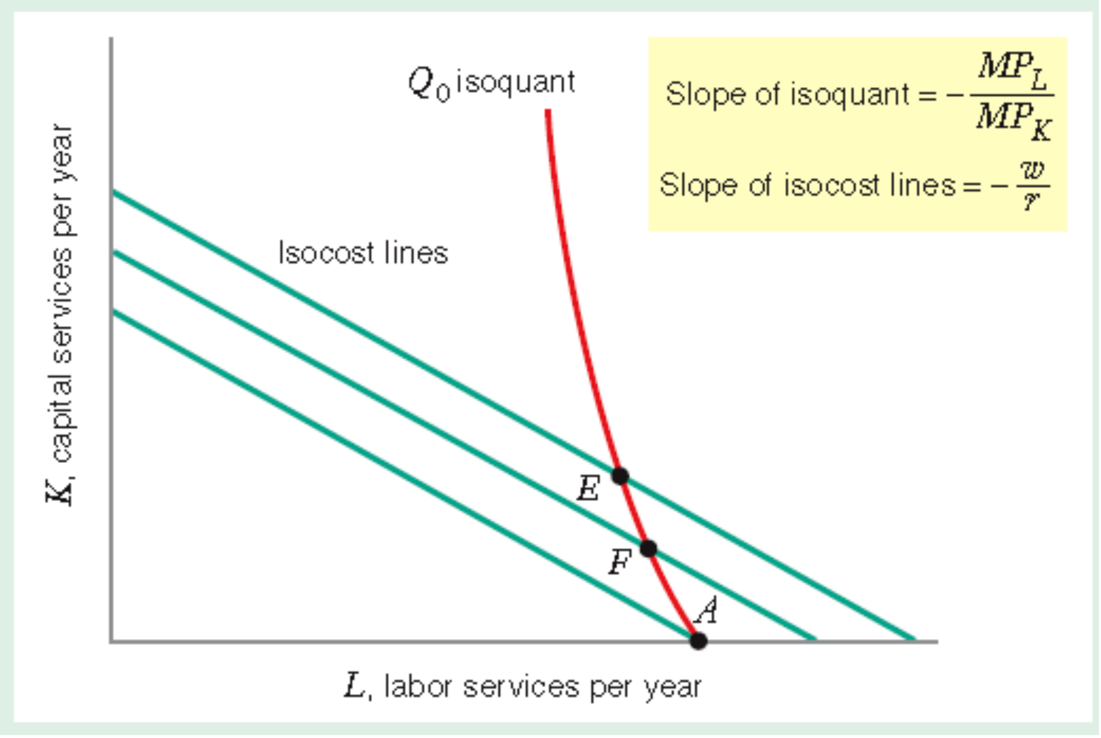
$$MRTS_{LK} = \frac{MP_L}{MP_K} = \frac{p_L}{p_K} \quad \text{or} \quad \frac{MP_L}{p_L} = \frac{MP_K}{p_K}$$

- Ratio denotes output in physical units over price
- When combination of inputs is efficient, the marginal product obtained from extra spending is the same for both inputs

# 2. Costs

**(Corner solutions)**

- Max/Min problem have same solution



# 2. Costs

Definitions:

- Opportunity cost of a resource is the value of that resource in its best alternative use.

  Example: £100 in facilities yields £800

  £100 in R&D yields £1000 revenue

  Opportunity cost of investing in facilities = £1000

  Opportunity cost if investing in R&D = £ 800

Opportunity cost depends on how we specify alternatives.

- Sunk cost: are costs that must be incurred no matter what the decision is. These costs are not part of opportunity costs.

  Example: It costs £ 5M to build a factory and has no alternative uses. £5M is sunk cost for the decision of whether to operate or shut down the factory

# 2. Cost minimisation problem

In the **SHORT-RUN…**
Suppose that one factor (say, K) is fixed.

The firm's short-run cost minimization problem is to choose quantities of the variable inputs so as to minimize total costs…
given that the firm wants to produce an output level $Q_0$…

and under the constraint that the quantities of the fixed factors do not change.

# 2. Cost minimisation problem

In the LONG-RUN…

$$\begin{Bmatrix} \underset{L,K}{Min} & p_L L + p_k K \\ st & Q(L,K) = Q \end{Bmatrix}$$

*Note:* L, K are the **variable inputs** and

$p_L L + p_k K$ is the **total variable cost**

Constraint: $Q(L,K) = Q$

# 2. Cost minimisation problem

In the SHORT-RUN…

$$\begin{cases} \underset{L}{Min} & p_L L + p_k \overline{K} \\ st & Q(L, \overline{K}) = Q \end{cases}$$

*Note:* L are the **variable inputs** and

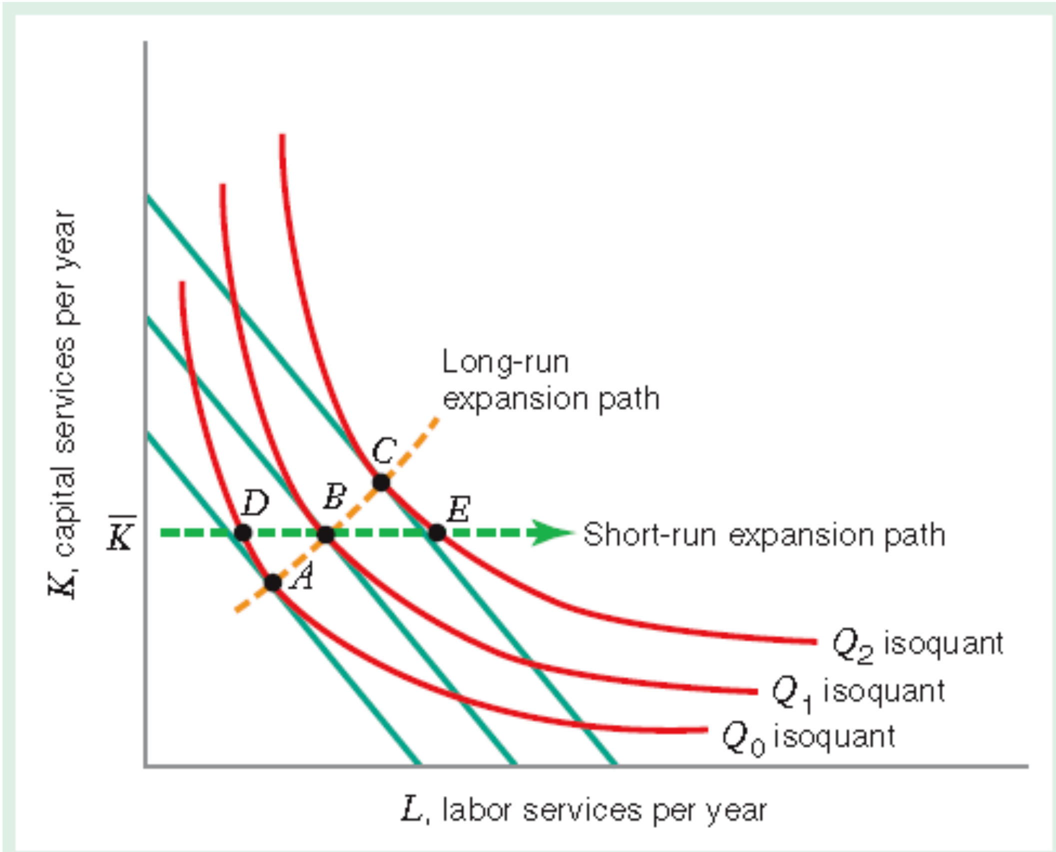
$p_L L$ is the **total variable cost**

$\overline{K}$ is the **fixed input and**

$p_k \overline{K}$ is the **total fixed cost**

Constraint: $Q(L, \overline{K}) = Q$

# 2. Cost minimisation problem



# 3. Cost Function

•LR average cost function is the long run total cost function divided by output, Q.

That is, the LRAC function tells us the firm's cost per unit of output…

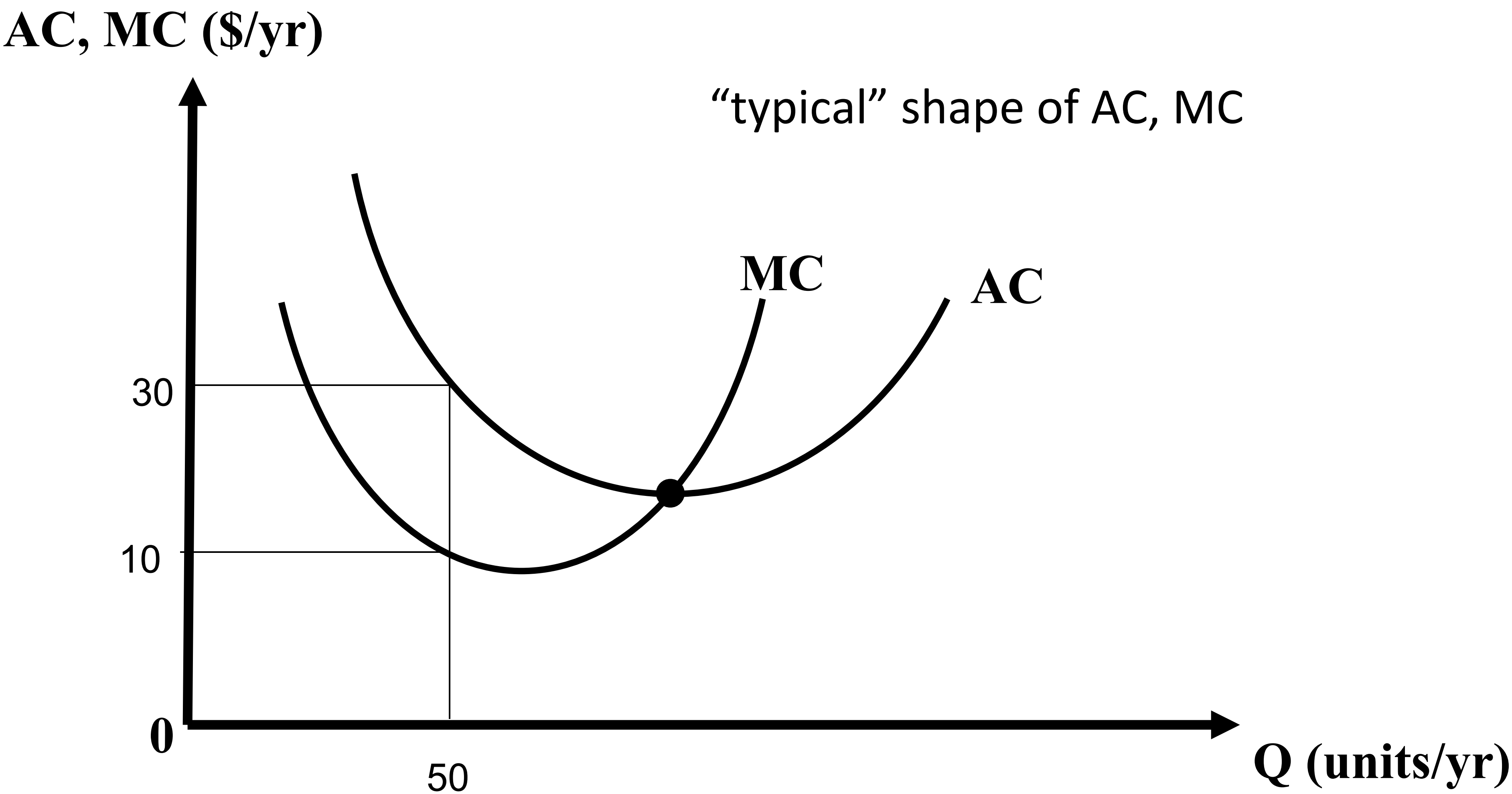
$$AC(Q,p_L, p_K) = TC(Q,p_L, p_K)/Q$$

•LR marginal cost function measures the rate of change of total cost as output varies, holding constant input prices.

$$MC(Q,p_L, p_K) = dTC(Q,p_L, p_K)/dQ$$

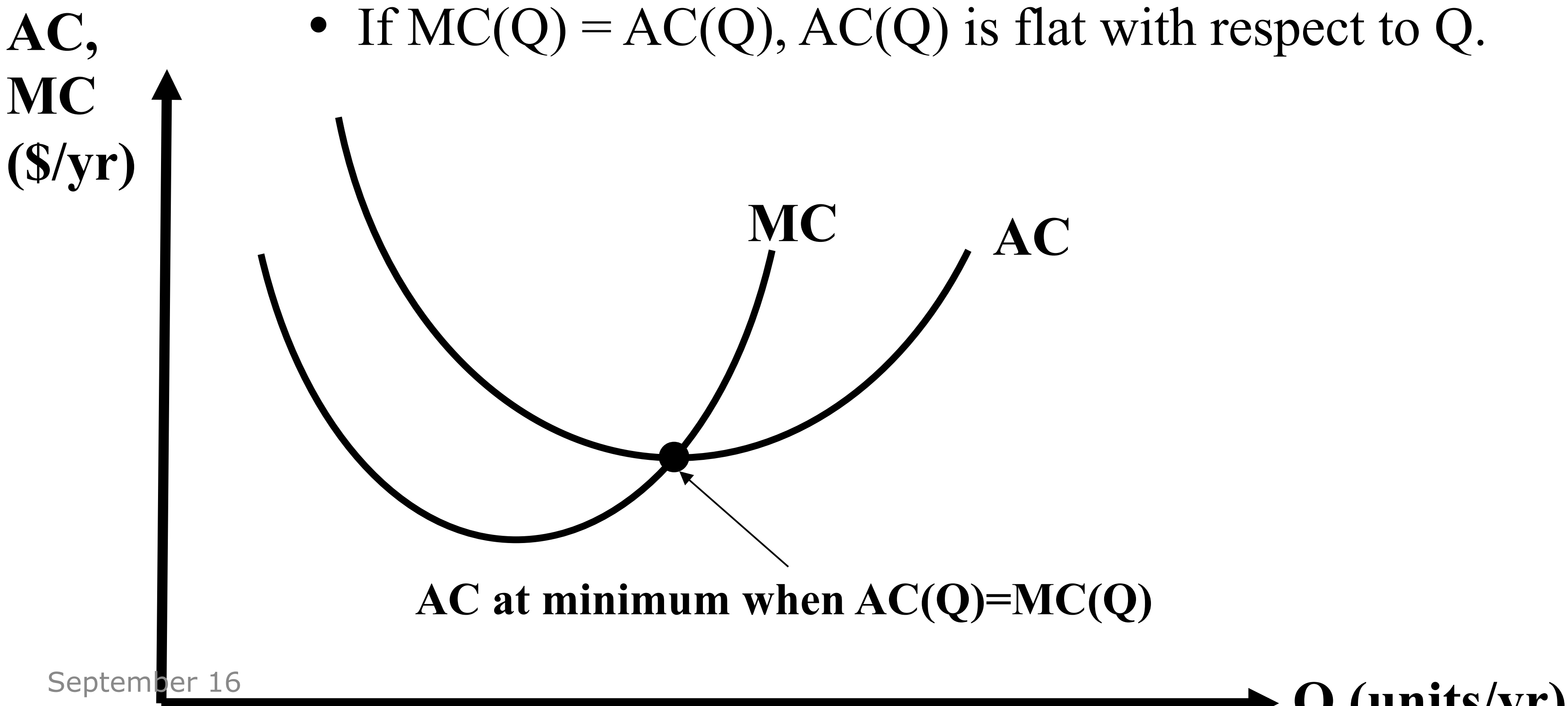
where: $p_L$, $p_K$ and constant

# 3. Cost Function



# 3. Cost Function

- Relationship between AC and MC
  - *Suppose that w and r are fixed…*
    - If MC(Q) < AC(Q), AC(Q) decreases in Q.
    - If MC(Q) > AC(Q), AC(Q) increases in Q.
    - If MC(Q) = AC(Q), AC(Q) is flat with respect to Q.



# 3. Cost Function

In the SHORT-RUN the total cost function tells us the minimized total cost of producing Q units of output, when (at least) one input is fixed at a particular level.

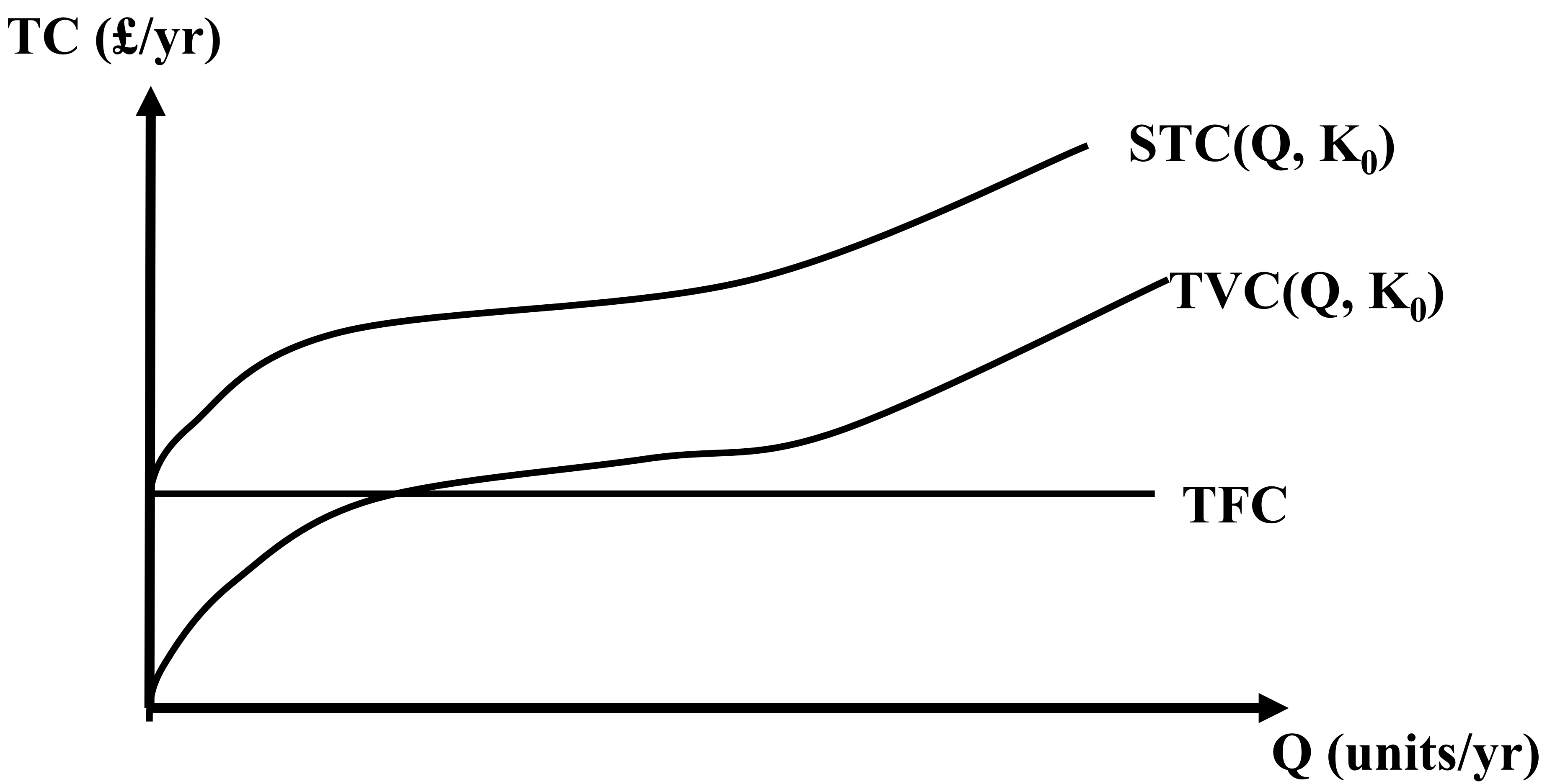
$STC(Q,K_0) = TVC(Q,K_0) + TFC(Q,K_0)$

Total variable cost (TVC) function is the minimized sum of expenditures on variable inputs at the short run cost minimizing input combinations.

Total fixed cost (TFC) function is a constant equal to the cost of the fixed input(s).

# 3. Cost Function

SR Total Cost, Total Variable Cost and Total Fixed Cost



# 3. Cost Function

Long-run Average Cost Function as envelope curve around the set of short-run average cost curves